A NEW MATHEMATICAL MODEL OF PRE-INDUSTRIAL DEMOGRAPHIC CYCLE

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The proposed model combines positive aspects of earlier models that have not been earlier combined within one model: it accounts for significant intercycle periods, on the other hand, through inclusion of the famine relief subsystem dynamics into the model it accounts for lengthy periods of very slow and unsteady population growth (when most part of the population had inadequate per capita acreage and there existed very strong incentives to innovate in the raising of the carrying capacity of land). Hence, this model could be used as a basis for development of a new generation of models accounting both for "secular cycles" and "millennial trends".

One of the most important recent findings in the study of the long-term dynamic social processes was the discovery of the demographic cycles as a basic feature of complex agrarian systems' dynamics. The presence of demographic cycles in the pre-modern history of Europe and China has been known since quite a long time, and already in 1980s more or less developed mathematical models of demographic cycles started to be produced (first of all for Chinese "dynastic cycles") [1]. At the moment we have a very considerable number of such models [2; 3; 4; 5; 6; 7; 8; 9]).

The focus of the present work is not to fit the existing historical data, with all the intricate features. Therefore, we do not feed in the model external warfare or temperature variables. Instead, we would like to mimic the qualitative behavior of the system in order to see whether the historical dynamics is consistent with verbal explanations offered by various authors. We would like to understand in what way annual climatic variations interact with population density to produce a demographic collapse through an increased frequency of internal warfare. We would also like to see how the country gets out of the state of disarray, and what

factors influence this "intercycle" period. Internal warfare and its inertia will play an important part in this model.

The only exogenous variable in our model is a fluctuating climate (that is, a fluctuating harvest yield). We will run our model and observe how cycles are formed, and what influences their period and amplitude. We set up the model as a difference equation where the value of the variable in a certain year is defined by the state of the system in the previous year. The model itself is presented below:

Harvest = H_0 + random fluctuations
U is "internal warfare inertia coefficient"
$U = 1/2(1 - \tanh[q(R_i - r)])$
$Food = Harvest*Area_i - seed$
dF is "food surplus" (which could be not only positive, but also negative)
$dF = Food - Food_{min}$

$dS = N_i*tax*dF$ if there is additional product
N_i*dF if there is shortage of food and enough storage
$-S_i$ if there is considerable shortage of food.

dR_i is number peasants turned into bandits in year *i*

$dR_i = -N_i*dF/Food_{min}*\alpha_{out}$ if there is not enough food
0 if there is enough food

α_{out} is "peasant-bandit transformation rate"

TaxFactor = U if there is additional product
0 if there is shortage

$Area_{i+1} = Area_i + U*(A_{total}/N_i - Area_i)$

N_i is the number of peasants
R_i is the number of bandits
S_i is the amount of food storage

$S_{i+1}=S_i + dS*TaxFactor$
$N_{i+1}=N_i*(1 + \alpha*dF) - dR_i - rob*N_i*R_i$
$R_{i+1}=R_i*(1 - \beta)+dR_i$

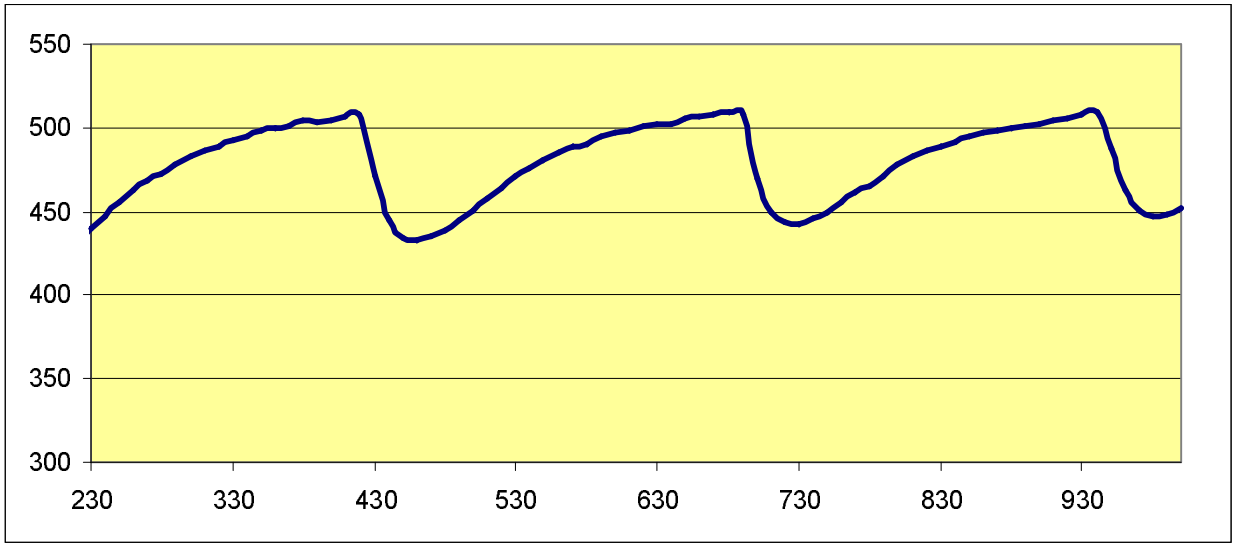
α is growth rate of peasants
β is death rate of bandits
rob – bandit-related death rate of peasants

We follow Chu and Lee (1994) in denoting as "bandits" both bandits and rebels (note that Chu and Lee in their turn just follow the Chinese tradition to

denote both categories with one term [*fěizéi*]). The logic behind this merging is that both groups produce rather similar effects on population dynamics: rebels might even fight with aim to improve the life of peasants, however, in order to feed themselves they would still have to take food from peasants. The presence of the state-sponsored relief system may seem to be too China-specific. Indeed, any developed systems of this kind are very rarely found within complex agrarian systems outside East Asia. However, in fact, some famine-relief counter-crisis sub-systems are found within most complex preindustrial states. Indeed, developed state-sponsored systems were found quite rarely; however, almost all complex agrarian states still possessed some counter-crisis subsystems. The most wide-spread type was based on the private stores of food resources held by elites (landowners etc.). During famines the elites would tend to use this storages to provide a sort of relief to at least some categories of commoner population. For example, landowners would not be normally interested in their tenants dying out and would provide some support to them in such cases. Such relief was very rarely quite altruistic. Thus landowners would provide some food to peasants in order to indenture them and to get their land; however, this would still help substantial parts of affected population to survive through lean years. For the sake of simplicity in our model both main types of pre-industrial counter-crisis relief subsystems are merged in one mechanism (hence, taxes are merged with rents).

This model generates the following dynamics (see Fig. 1):

Fig. 1. Populations Dynamics Shape Generated by the Model



The proposed model combines positive aspects of earlier models that have not been earlier combined within one model. Unlike Nefedov's model, but like some prey-predator logic based models it accounts for significant intercycle periods. On the other hand, unlike the latter, through inclusion of the famine relief subsystem dynamics into the model it accounts for lengthy periods of very slow and unsteady population growth (when most part of the population had inadequate per capita acreage and there existed very strong incentives to innovate in the raising of the carrying capacity of land). Hence, this model could be used as a basis for development of a new generation of models accounting both for "secular cycles" and "millennial trends". In order to do this, we suggest in future to alter basic assumptions of the earlier generations of demographic cycle models (that both the carrying capacity of land and the polity size are constant). These are variables with long terms trend dynamics in the rise of carrying capacity of land, cultural complexity, and empire sizes that the new generation of models needs to account for. Demographic cycle models account at present only for cyclical dynamics; the new modeling should be able to interconnect trend with phase dynamics.

There are both theoretical and empirical grounds to maintain that the carrying capacity of land not only experienced a upward long-term trend, but also

that the innovations contributing to this trend occurred in particular phases of demographic-political cycles. Incentives created by relative abundance of resources in the initial upward growth phases of agrarian state demographic-political cycles tend to be insufficient to create the innovations leading to the rise of the carrying capacity of land (these phases, however, were also very important, as during them there were strong incentives for the innovations leading to the rise of productivity of labor). Rather, innovations raising the carrying capacity of land tended to occur during intermediate growth before the demographic collapse phase. While these innovations usually acted only to delay demographic collapses, they secured the existence of a very important upward trend, which could be to some extent accounted for as a by-product of the demographic cycles.

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